1853MNRAS..13..188L

On the Mathematical Theory and Practical Defects of Clock Escapements, with a Description of a new Escapement, &c. By James M'Kenzie Bloxam, Esq.

In this communication the author gives a very complete theory of the dead beat and the remontoire escapements, and shows that in some cases the remontoire is preferable. He also describes a remontoire gravity escapement, which appears to possess some peculiar advantages.

On the Classification of Comets and the Distribution of their Orbits in Space. By Dr. Lardner.

The activity and zeal of observers and computers have of late years supplied vast additions to the knowledge of cometary astronomy. The elements of the orbits through which these bodies have moved through the system have been especially multiplied, and it might be naturally expected that if their orbits like those of the planets be regulated by any particular law in relation to their positions in space, either with reference to each other or with reference to any invariable plane passing through the centre of the sun, such law would be discoverable. It cannot fail, therefore, to be both interesting and useful to review and compare the numerous data thus supplied, with the especial view to the discovery of a law governing the position of the cometary orbits in space, if any such law exist; and if no such law exist, to demonstrate the remarkable fact that comets move in all conceivable directions, and in all conceivable planes, without any greater prevalence in any one direction or in any one plane than in any other.

The catalogues of Pingré and his contemporaries have been of late considerably enlarged by German astronomers, and a very complete one was published within the last few years by Dr. Mädler, in his elementary work on astronomy. This, however, has been lately continued to a much more recent date, and enlarged by the interpolation of several ancient comets, whose elements have been recovered from various traditions and historical records by the zeal and activity of Mr. Hind, to whom science is so largely indebted for his discoveries in this and other branches of practical astronomy.

Taking this enlarged catalogue of Mr. Hind as the most ample repertory extant of cometary data, we find that we are in possession of the elements of the motions of 207 comets. It is true that those which appeared before the middle of the seventeenth century can only be regarded as approximations, some of which may vary from the truth within considerably wide limits, and a few of the comets observed since that date have appeared for such periods, and under such disadvantageous conditions for observation as to supply rather imperfect data; still, on the whole, it may be considered that we have before us the elements more or less exact of the orbits of 207 comets, whereon to base inductive reasoning directed to the discovery of the laws which govern their position, if any such exist.

Of these 207 comets, it appears that 40 move in ellipses, 7 in hyberbolas, and 160 in parabolas.

The 40 comets to which elliptic orbits have been assigned may be conveniently classified in three groups, as follows:—

1st group — 13 comets, whose aphelia are included within *Saturn's* orbit.

2d group — 6 comets, whose mean distances are nearly equal to that of *Uranus*.

3d group — 21 comets, whose mean distances exceed in a greater or less proportion the extreme limits of the solar system.

First group of elliptic comets.—The 13 comets composing this group are as follows:—Encke, Biela, Faye, De Vico, Brorsen, D'Arrest, Clausen (1743), Burckhardt (1776), Lexell (1770), Blainpain (1819), Pons (1819), Pigott (1783), Peters (1846).

By reference to Mr. Hind's table, it will be perceived that these 13 comets retain in a striking manner many of the characteristics of the planetary motions; they are all, without a single exception, *direct;* their inclinations (with one single exception) are all included within the planetary limits, and even the exceptional comet does not much exceed the inclination of *Pallas;* the excentricities, though of an order of magnitude greatly exceeding even that of *Juno* (the most excentric of the planets), still are incomparably less than those of the other comets. While the aphelia of these orbits generally lie a little within or without the orbit of *Jupiter*, their perihelia are included between the orbits of *Mars* and *Mercury*, the only exception to this being Peters' comet, whose perihelion lies beyond the orbit of *Mars*, while the aphelion extends to the orbit of *Saturn*.

Second group.—Comets of Halley, Pons (1812), Olbers (1846), De Vico (1846), Brorsen (1847), Westphal (1852).

Of the 6 comets which compose this group, 5 have direct motion, that of Halley alone being retrograde. The orbits, however, in all other respects differ in a striking manner from those of the first group, and retain no other trace of resemblance to the planetary orbits. The excentricities are great compared with those of the first group, and of an order to efface all analogy whatever with the planetary ellipses. Their inclinations also range far beyond the planetary limits.

It would, therefore, appear, that this group holds an intermediate place between the first group and the more numerous classes which we are now about to notice.

Third group.—Comets of 1680, 1683, 1763, 1769, 1780, 1793, 1807, 1811 (A), 1811 (B), 1822, 1825, 1827, 1830, 1840 (A), 1840 (B), 1843, 1844, 1845, 1846 (A), 1846 (B), 1849.

Of the 21 comets composing this group, 11 are retrograde and 10 direct.

It appears, therefore, that the tendency to direct motion, which

1853MNRAS..13..188L

prevails with a single exception in the first two groups, is entirely effaced in the last; no tendency, on the other hand, however, being discoverable towards the other direction, since the proportion of 11 to 10 in favour of retrograde motion must be regarded as casual.

Fourth group.—Comets of 1729, 1771, 1774, 1818, 1824, 1840, 1843.

Of the 7 hyperbolic comets composing this group, 6 are direct; but among their orbits there appears to be no other prevalent character.

Fifth group.—This group consist of 160 comets, whose orbits are sensibly parabolic.

The distribution of these bodies with relation to the direction of their motion is as follows:—

Direct	70
Retrograde	86
Unascertained	4
	160

This would indicate a slight tendency in favour of retrograde motion, but if we take into account the entire number of comets whose orbits are ascertained this tendency will then disappear, for we shall then have,

Direct	94
Retrograde	99
Unascertained	4
	207

This difference in favour of retrograde motion must be considered as casual, and, so far as the data now before us extend, it must be inferred that the comets taken generally, unlike the planets, show no prevailing tendency either to direct or retrograde motion, but that the elliptic comets of the smaller excentricities show a decided tendency to direct motion.

To determine whether the planes of the cometary orbits are characterised by any prevailing tendency towards a particular inclination, it will only be necessary to ascertain the number of orbits whose inclinations are included between certain definite limits.

It appears from a table which I have formed, exhibiting the proportion of comets in every 100, whose inclinations are included within every successive 10° of the quadrant, that the planes of the orbits of the comets have a decided tendency to crowd around an inclination of about 45°, or to take positions which would be those of tangent planes to a right cone, whose axis would be at right angles to the plane of the ecliptic, and whose semi-angle would be 45°.

That this tendency is too marked to be considered as casual, will be admitted when it is considered that the mean number of inclinations per cent due to each 10° would be 11, while the number of inclinations included between 40° and 50° is nearly 16, being nearly 50 per cent greater than the mean.

1853MNRAS..13..188L

To determine, in like manner, whether the nodes and perihelia exhibit any preference for any particular longitude, I have constructed a table giving the number of nodes and the number of perihelia in every 100 included within every 30° of longitude, from 0° to 360°.

From this table it would appear that there is a tendency in the nodes to crowd into the third and seventh signs of the ecliptic. The mean number of nodes due to each sign being 8·33, the number of the third sign is very nearly 12, and the number in the seventh nearly 10; the former being nearly 50 per cent and the latter 20 per cent above the mean.

The perihelia, in the same manner, show a tendency to crowd into the third, fourth, and tenth signs, the numbers in the third and fourth being 12, and the number in the tenth nearly 13, each being about 50 per cent above the mean.

It is difficult to conceive, when the large number of comets upon which these calculations have been based, is considered, that these results, either as regards the perihelia or the nodes, can be casual.

The distribution of the points where the comets arrive at perihelion, with reference to their distances from the sun, supply another ground of classification.

This object may be effected by the formation of a table, exhibiting the number of comets in every 100 which have their perihelia between the surfaces of spheres distributed round the sun, with the radii of 20, 40, 60, &c. millions of miles.

It appears from such a table that only a small proportion of the perihelia lie outside the sphere, whose radius is 120 millions of miles. This circumstance must be ascribed to the fact, that comets moving in such orbits will mostly escape observation; but it may, perhaps, be assumed that, *cæteris paribus*, the comets, whose perihelia lie within a sphere through the earth's orbit, have nearly equal chances of being observed. If this be assumed, then it will follow that the numbers of such comets which have been observed are nearly proportioned to their total numbers; and, therefore, that the numbers within this limit in the preceding table do actually represent approximately the distribution of the points of perihelion round the sun.

If we compare, then, the number of perihelia situate between the equi-distant spheres indicated in the table above referred to, with the cubical spaces through which they are respectively distributed, we shall obtain an approximate estimate of the *density* of their distribution in relation to the distances from the sun. I have computed the following table with this view. In the second column I have given the number of comets per cent, whose perihelia are included between the equi-distant spheres: in the third column, the numbers express the cubical spaces between sphere and sphere, the volume of the sphere, whose radius is 20 millions of miles, being the cubical unit; and in the fourth column the numbers are the quotients of those in the second, divided by those in the third; and,

1853MNRAS..13..192G

therefore, express the successive densities of the perihelia between sphere and sphere.

	Numbers of Perihelia.	Cubical Space.	Density of Perihelia.
0 — 20	8·65	1	8·65
20 40	11·70	7	1·67
40 60	20·30	19	1·06
60 80	17·20	37	0·47
80 100	20·80	61	0·34
100 120	8·65	91	0·095

It is evident, then, that the density of the perihelia increases rapidly in approaching the sun. If the numbers in the last column of the table be compared with the inverse powers of the distance, it will be found that this increase of density is more rapid than the inverse distance, but less so than the inverse distance squared.

Two planes being supposed to be drawn, each at an angle of 45° with the plane of the ecliptic, and so that the intersection of one of them with the plane of the ecliptic is directed to the 75° of longitude, and that of the other to the 200° of longitude, there is a tendency of the cometary orbits to crowd towards the directions of these planes, not certainly so decided as to establish a law, but too pronounced to be casual.

Note on the Correction to be applied to the Apparent Ellipticity of a Planet, in consequence of the Elevation of the Earth above the Plane of the Planet's Equator. By Mr. Grant.

As the subject of the ellipticity of the planets, more especially *Saturn*, appears to excite considerable interest at present among astronomers, it may be desirable to exhibit a convenient mode of computing the effect due to the elevation of the earth above the plane of the planet's equator.

Let A B A′ B′ represent an elliptical section of the planet formed by a plane passing through the polar axis B B′ and the eye of an